**ME-Project Report**

**about**

**Quad-rotors & Payload system dynamics and stability**

By: Ran , Oct 2017

# Abstract

This report is the summary of my research work as part of Master of Engineering in Autonomous Systems and Robotics ( [TASP](http://tasp.technion.ac.il/) ).

In this research, I am investigating the stability of payload carried by a 2 quadrotors ‘array’, under certain conditions. This case is in interest because of possible payload delivery mission required by companies such ‘Amazon’ and others, to deliver a relatively big-size and heavy payload – a mission that is not always possible for 1 quadrotor alone.

I’ll show the model equations of motion, analytical investigation and a numerical investigation results for comparison.

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# Introduction

There is a growing amount of interest in the controlled autonomous behavior of collectively operating unmanned aerial vehicles. An example is an array of autonomous quadrotors which are under consideration for a variety of missions including surveillance [Acevedo et al., 2012], heavy payload delivery [Bernard et al, 2011], and assembly of structures [Kumar and Michael, 2012]. While there is an abundance of documentation of multi-agent behavior of very large groups (or swarms) such as flocks of birds and schools of fish which can quickly adapt to a complex terrain [Shklarsh et al, 2011] or environmental conditions [Elor and Bruckstein, 2011; Agmon et al, 2011], and an uncertain and changing environment in nature [Young et al, 2013], there is limited research on small size arrays of autonomous elements that continue to maneuver collectively under severe environmental conditions.

Documented research on single quadrotor dynamics, stability and control consists of rigid-body dynamical systems models augmented by angular rotor dynamics [Leishman 2006]. Investigations include nonlinear control for take-off, hovering and landing [Kendoul et al, 2007] and to overcome path following uncertainties [Raffo et al, 2010] and disturbances [Schoellig et al., 2012]. Control of aggressive maneuvers such as flying through a narrow gap [Mellinger et al, 2012] have been implemented, and investigations include robustness analysis applied to wind gusts [Alexis et al, 2011; Escareno et al, 2013]. Recently, control of cable suspended payload has been proposed [Sreenath et al, 2013].

Evasive steering for collision avoidance is essential in both swarms [Ribak et al, 2012] and arrays of unmanned aerial and space vehicles [Mazal and Gurfil, 2013]. However, to-date the focus of research on collective behavior of multiple quadrotors has been on indoor operation in a variety of arenas [Hoffman et al, 2011; Kumar 2012; D’Andrea 2013]. A multiple quadrotor model has been recently proposed and investigated numerically to yield trajectories for a three element array carrying a payload [Sreenath et al, 2012]. An additional example is that of a multi-agent distributed flight array which incorporated multiple modular single-rotor aerial vehicles capable of autonomous assembly (docking) and coordinated flight, modeled as hybrid dynamical system was simulated numerically and experimentally [Oung and D’Andrea, 2011].

Motivated by simulation studies of decision making in animal groups in motion, the stability of multiagent particle dynamical systems models have been analyzed to reveal cohesive behavior [Liu and Passino, 2005] and separation of fast and slow time scales reflecting a local bifurcation structure indicative of a compromise by individual elements with conflicting preferences [Nabet et al., 2009]. Symmetrical and asymmetrical bifurcations have been shown in a swarm robotics test bed [Garnier et al, 2013] and a noise intensity threshold was shown to govern swarm transition from a misaligned state into an aligned state [Mier et al, 2012]. Furthermore, nonlinear multi-agent swarm models exhibit existence of periodic limit-cycles culminating with non-stationary chaotic solutions [Das et al, 2012], and stochastic bifurcations [Ebeling and Schimansky-Geier, 2008].

In the light of the current scientific background the behavior in severe environmental conditions is yet unresolved. Thus, our general aim is to derive a consistent dynamical systems model for a small size array of quadrotors which can withstand severe and unsteady aerodynamic disturbances. We propose to investigate the nonlinear array dynamics asymptotically and numerically, culminating with a system bifurcation structure highlighting orbital stability thresholds. The significance of research is thus twofold: i) the proposed combined analytical and numerical methodologies will enable identification of instabilities for both single and a small size quadrotor array in severe conditions, and ii) construction of a nonlinear stabilization strategy for a small size array will yield conditions for synchronous operation that can autonomously withstand unsteady aerodynamic conditions. This combined approach is original and is anticipated to bridge the gap between documented indoor operation and large time-dependent perturbations expected in a changing non-stationary environment.

The paper includes : i) derivation of a theoretical nonlinear dynamical systems model for a single quadrotor in unsteady conditions [Leishamn, 2006] including model-based estimation of the corresponding nonlinear system properties [Gottlieb and Habib 2012]. Ii) The single element model will then be extended to a small size array incorporating selected constraints including adaptive maneuvering and collision avoidance, iii) the dynamical systems model for the quadrotor array will be investigated via the asymptotic multiple-scales method [Kevorkian and Cole 1995] to yield stability thresholds for synchronous and non-stationary dynamics [Gutschmidt and Gottlieb, 2012], iv) numerical orbital stability analysis of the dynamical system will be employed to validate the asymptotic stability thresholds [Nayfeh and Balachandran, 2005; Aginsky and Gottlieb, 2012], v) stabilization of system response subject to severe conditions [Nijmeijer and van der Schaft 2010].

Bkg

motivation

literature survey

objectives

report structure

In this paper, I will describe the dynamical system of 2 Quad-Rotor (aka quad) units, utilizing a common payload.

The problem formulation assumes 2D framework. The more general 3D case is not treated here.

The quads motion is treated as system inputs, and not discussed here by itself. I will discuss the payloads’ dynamics and stability.

The investigation work flow will be:

1. The dynamic equations of the quads and payload will be described, and some limiting cases will be shown to verify the model.
   1. Coordinates definition in inertial frame
   2. Lagrangian term composition
   3. Deriving the equations of motion without non-conservative forces
   4. Verify result with limiting cases of:
      1. Elastic pendulum
   5. Find natural frequency, from equilibrium state
   6. Referring to non-conservative forces (and moments)
   7. Move to non-dimensional terms (by length and time scales)
   8. Define the treated maneuver in the problem (hover, translation of payload from points A to B)
2. Characterize the problem with certain parameters. Such as (, , ) and initial conditions and maneuvers ()
3. The next step in this work will be to analyze the equations by Multiple Scales method or Averaging method.

# Nomenclature

i : index for object {1,2,p} regarding: quad #1, quad #2, Payload, or: cable #1, cable #2.

: returning force constant of the linear spring i

: free spring length (not loaded)

: current length of the loaded spring

: mass of object i

: location of the i’th mass center, in inertial coor.system

: rotation angle around axis, of the rigid body payload, relative to the Inertial frame.

: geometric length of the payload rigid body

: geometric height of the payload rigid body

: rotation matrix from payload to Inertial coordinate frame

: rotation matrix from Inertial to payload coordinate frame

: moment of inertia, around axis , for object i

L : Lagrangian of the system

T : kinetic energy

V : potential energy

u : air velocity in global framework, in X direction

v : air velocity in global framework, in Y direction

: air density. Treated as constant

: drag coefficient of the payload. Taken as equal for both directions of X, Y.

Acronyms

DOF : Degreed of Freedom

EOM : Equations of motion

# Problem formulation and system dynamics

The examined system is composed of 2 units of quadrotors, and 1 rigid body payload which is connected to each of the quadrotors by cables connected to the anchor points.

The system is described and investigated in the 2D world.

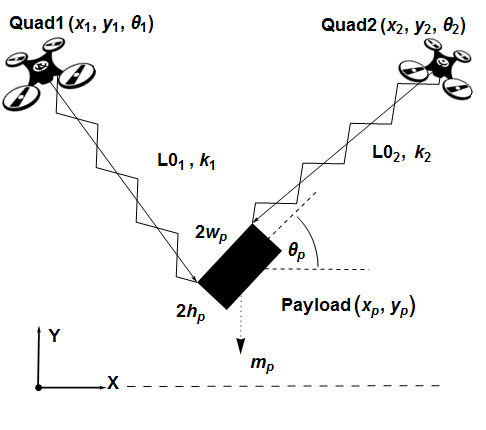


Figure - system setup

**Some assumptions are being made and they are:**

Quadrotor:

1. Quad body and parts are **rigid**. *No* elasticity is included.
2. Geometry structure is **symmetrical** in relation to the principal axes. And the mass distribution is **uniform**. Hence the Inertia matrix is taken as pure diagonal.
3. **quads motion is treated as** **given**! Hence the quad pure dynamics and control are not considered here, and they are mentioned only to give a better perspective for the problem in hand.

Payload & cable construction:

1. The ‘cable’ which the payload is connected to, is modeled as a **lumped mass linear spring**, with initial length , and a linear returning force coefficient .
2. The cable is connected to the quadrotor exactly in its center of mass (C.G).

There is no friction and moment actuated through those hanging points.

1. In the payload hanging points there is an arbitrary **structural dumping** (resultant of hanging point friction and spring dumping).
2. The payload is a **rectangular box**, similar to a ‘CONEX’ cargo container, characterized by width of (, and height of (, and with inertia matrix .
3. There is no consideration in possible aerodynamic drag of the cables.
4. **Simplified aerodynamic forces** (lift and drag) on the payload are considered – will be addressed in the non-conservative forces section.

**Coordinate systems , State variables, and Rotation matrices**

I – inertial coordinates frame. It is the global reference point for the problem.

Its’ axes are :

P – Payload coordinate frame. The origin is located at the C.G of that rigid body.

Since we are looking at 2D problem, for each system element we have 3D.O.F which are planar position and pitch angle. For the full 2D problem, we have the general coordinates as:

(1)

which is a 9 D.O.F problem.

The problem geometry

Schematics of the system, in accordance with the nomenclature listed above:

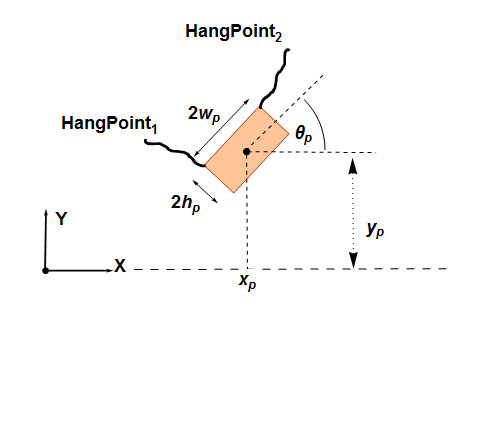


Figure - detailed sketch for hanging points geometry

Where

(2)

The Lagrangian of the system is:

By definition of the Lagrangian , we construct the system Lagrangian:

(3)

Where

The Lagrange equations will be calculated using the next equation (while using the common fact that V is not dependent on for mechanical systems):

(4)

– are the non-conservative forces.

*Note:* If I would consider the inner dynamics of each of the quads and their controllers – I would need to consider at least those 9 Equations Of Motion.

Because this is a ‘Research project’ and not a full research work - I will focus my desired analysis on the payload stability, and I will limit myself to deal with , while () are treated as given.

The dimensional 9 EOM are described in Appendix A for reference.

non-conservative forces:

we consider the next general forces (for the payload-related general coordinates):

- *structural dumping*

As described briefly in the assumptions section above. It is related to the relative movement of the payload to each of the quadrotors, and will be written as:

; (5)

- *aerodynamic forces*

While the desire is to use simple terms for the aerodynamic contribution, I will assume just drag forces in x,y directions. Furthermore, I’ll assume which is like ones’ of ‘CONEX’ cargo container.

(6)

u,v are the air velocities in the relevant directions.

Equations of Motion (dimensional):

As mentioned above, from now on we deal with only 3D.O.F – for the general coordinates of the payload.

From (4), using (3),(5),(6), and rearranging, we can get the *dimensional* equations of motion for the payload:

(7)

Where:

(8)

\* *note*: ‘c’ is for Cos(), ‘s’ is for Sin()

Non-dimensional Equations of Motion:

Scaling the dimensional variables according to these relations:

; (9)

This results also in:

;

So **the non-dimensional equations** can be written in matrix form in the following way:

**(10)**

Where ( :

; ;

;

;

; ; ;

further analysis assumptions:

in order to simplify the analysis continuation and to focus on the method, I’ll assume the geometry of the system is symmetric. Hence: , =1 from here next.

Moreover (around the payloads’ Z axis, in center of gravity) is the one relevant for a rectangular shape which is:

(11)

I’ll check only for the case where quadrotors inputs are:

(12)

Limiting case test of elastic pendulum is shown in Appendix 1.

# Equilibrium analysis

Equilibrium points:

We find the equilibrium possible states by setting all time derivatives to zero ().

It results with all accelerations, aerodynamic forces, and dumping forces and moments are zeroed.

The solutions are:

**;**  (13)

* From now on I’ll relate only to because this is the case of the quads can take the payload. The 2nd case can be related to similar case of inverted pendulum.

Linearizing about that equilibrium point:

The goal now is to find the natural frequencies of the system. it is done by looking on the effect of small deviations around the equilibrium point, while the aerodynamic and dumping forces are zeroed. Using perturbations syntax:

; ; (14)

Detailed derivation of the equations is in Appendix B.

From this derivation, we see that the natural frequencies of the system are dependent and cannot be separated for specific general coordinate. We sign them as .

# Asymptotic analysis

\*for selected limiting cases that reveal a Hopf bifurcation and/or an orbital instability

## Maneuvers

Planned maneuvers to test the system are:

1. hover
2. translation of payload from points A to B, in a straight line. With equal or different quads heights.

Trajectory can be described for example as:



(\*)

# Numerical analysis

\*for asymptotic validation vs general maneuver

# Discussion

# Summary

I described the 2D dynamics of system of 2 quadrotors and 1 connected rigid body payload.

I verified against limiting cases of:

1. elastic pendulum

Non-dimensional equations were submitted.

# Acknowledgements

Thanks for my instructor professor Oded Gottlieb for his patience and the detailed guidance through this work.

# References

1. Sadr, S. et al. “Dynamics Modeling and Control of a Quadrotor with Swing Load.” Journal of Robotics 2014.December (2014): 1–12. Journal of Robotics. Web.
2. Bouabdallah, S. “Design and Control of Quadrotors With Application To Autonomous Flying.” École Polytechnique Fédérale De Lausanne, À La Faculté Des Sciences Et Techniques De L’Ingénieur 3727.3727 (2007): 61.

Used articles about:

Slam as mission related (algs) (1 only)

Single quad dynamics (and some related PID or other controllers) (1 or 2)

Quad with sling load dynamics and control (stability. Trajectory planning) (2)

Quads or helicopter with common load payload (2)

Non linear dynamic analysis about quad dynamics ,

Transition to problem of rigid body hanged on two spring strings (with aerodynamics effect), where quads are just general inputs to the problem ->

Non linear analysis of simple pendulum

-“- of rigid body between 2 moving / oscilating plates

# Appendices

## Appendix 1 –Limiting case dynamics – elastic pendulum

Reminding about the full problem equations of motion, from (10):

When looking on elastic pendulum for lumped mass, we can assume:

1. for the lumped mass (hence doesn’t matter any more)
2. for the connection, only to the first base, and not the 2nd one (
3. Arbitrarily I will assume which means also the 1st base is static

The equations of motion become:

(1.1)

(1.2)

Finding the equilibrium point - we set the derivatives of to 0 :

(1.3)

The 1st option is the relevant one, for the considered state of .

We can also note that if considering : , which fits to a problem of a simple pendulum, hanged on a rod and not a spring.

assumption is yp>0 fits to , so otherwise it means the spring is to small and weak.

assumption is yp<0 fits to

### Linearization around the equilibrium point

**1st order linearization** is :

*Near equilibrium*:

Testing for the 1st equilibrium point of:

;

(1.4)

The equations are written as:

* Using (1.4) and neglecting small terms such as :

(1.5)

* Using the relation from equilibrium above to eliminate o(1) terms

(1.6)

It is equivalent to the matrix notation :

(1.7)

Where :

Where we know to find the natural frequencies by the requirement of:



It is an equation of 4th order for , the relevant solutions are:



, where **expecting**

If reformatting the above to it is equivalent to the natural frequency of simple pendulum with a constant length of !

*Extra Limiting cases:*

When it affects we get and which is similar to

## Appendix A –Equations of Motion for 9D.O.F case

From (4), using (3), we get the *dimensional* equations of motion for the 2 quads and the payload:

\* no non-conservative forces are included here.

Where:

## Appendix B –Linearized equations of motion

Reminding the non-dimensional equations from (10):

We set

; ; (B1)

; ;

While

; ; (B2)

Later on I’ll use small angles rounding:

(B3)

and neglecting multiplications of the small perturbations elements (i.e. , , etc.).

Remember that quads locations are also given, as in (12).

I would like to develop the terms by a Taylor series expansion.

(B4)

after small angle assumption is:

After implementing (B4) in (10) and eliminating more small elements multiplications:

Or setting in the matrix form of which gives:

(B5)

Where the natural frequencies are calculated from:

(B6)

We get 3 different natural frequencies. They are dependent.